BIBLIOGRAPHY

1. Krasnosel'skii M. A. Positive Solutions of the Operator Equations. M., Fizmatgiz, 1962.
2. Gershuni G. Z., Zhukovitskii E. M. and Iurkov Iu. S. On convective stability in the presence of periodically varying parameter. PMM Vol. 34 №3, 1970.
3. Zen'kovskaia S. M. and Simonenko I. B. Effect of the thigh frequency vibrations on the onset of convection., Izv. Akad. Nauk SSSR, MZhG, №5, 1966.
4. Zen'kovskaia S. M. Study of the convection in a fluid layer in the presence of vibrational forces., Izv. Akad. Nauk SSSR, MZhG, №1, 1968.
5. Simonenko I. B., Proof of the method of averaging for the abstract parabolic equations., Matem. sb., Vol. 18(123), № 1, 1970.
6. Simonenko I. B. Proof of the method of averaging for the problem of convection. In the collection: Mathematical Analysis and its Application, Izd. Rostovsk. Univ. Vol. 1, 1969.
7. Burde G. I. Quantitative investigation of convection arizing in a modulated external force field., Izv. Akad. Nauk SSSR, MZhG, №2, 1970.

Translated by L. K.

UDC 539.3

SOME CONTACT PROBLEMS FOR AN ELASTIC INFINITE WEDGE

PMM Vol. 36, №1, 1972, pp. 157-163

B. M. NULLER
(Leningrad)

(Received May 18, 1970)

Non-self-balanced homogeneous solutions of the mixed plane problem of elasticity theory for and infinite wedge $-\alpha \leqslant \theta \leqslant \alpha$, $0 \leqslant r < \infty$, one part of whose boundary $\theta = \pm\alpha$, $0 \leqslant r \leqslant 1$ is under the conditions of sliding constraint and the other is stress-free, are constructed and investigated. The solutions are of independent interest since they determined the state of stress of a wedge (a wedgelike strip in an elastic plane) on which a load equivalent to a longitudinal force P, a transverse force Q and a moment M (see Fig. 1) acts through a stiff yoke (a wedgelike stamp). Together with the statically balanced homogeneous solutions, they form a system of functions needed to solve mixed problems for elastic finite sectorial domains by the method elucidated in [1].

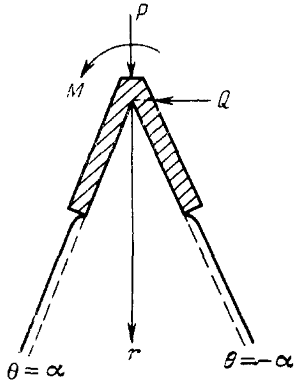

Fig. 1.

1. Symmetric problem. Let us write the condition on the wedge boundary for $\theta = \pm\alpha$:

$$u_{\theta*} = 0 \quad \text{for} \quad 0 \leqslant r \leqslant 1, \qquad \sigma_\theta = 0 \quad \text{for} \quad 1 < r < \infty \tag{1.1}$$

$$\tau_{r\theta} = 0 \quad \text{for} \quad 0 \leqslant r < \infty \tag{1.2}$$

$$\sigma_\theta \sim (1-r)^{\varepsilon-1} \quad \text{for} \quad r \to 1-0 \qquad (\varepsilon > 0) \tag{1.3}$$

In the Papkovich-Neuber formulas for the elastic displacements [2]

$$2Gu_r = \varkappa(\Phi_1\cos\theta + \Phi_2\sin\theta) - r\left(\cos\theta\frac{\partial\Phi_1}{\partial r} + \sin\theta\frac{\partial\Phi_2}{\partial r}\right) - \frac{\partial\Phi_0}{\partial r} \tag{1.4}$$
$$2Gu_\theta = \varkappa(\Phi_2\cos\theta - \Phi_1\sin\theta) - \left(\cos\theta\frac{\partial\Phi_1}{\partial\theta} + \sin\theta\frac{\partial\Phi_2}{\partial\theta}\right) - \frac{1}{r}\frac{\partial\Phi_0}{\partial\theta}$$

(G is the shear modulus, $\varkappa = 3-4\nu$ and ν is the Poisson ratio), let us set

$$\Phi_0 = 0,\ \Phi_1 = F_1,\ \Phi_2 = F_2 \tag{1.5}$$

$$F_1 = \frac{1}{2\pi i}\int_L [A_1(p) + A_2(p)]\cos p\theta r^{-p}dp,\quad F_2 = \frac{1}{2\pi i}\int_L [A_1(p) - A_2(p)]\sin p\theta r^{-p}dp \tag{1.6}$$

Let us select the path of integration L between the imaginary axis and the pole λ of the integrands closest to it in the half-plane $\operatorname{Re} p < 0$. Inserting the Mellin transforms for $\theta = \alpha$

$$\sigma^+(p) = \int_0^1 \sigma_\theta r^p dr,\qquad u^-(p) = \int_1^\infty u_\theta r^{p-1}dr \tag{1.7}$$

and utilizing their inverses, from conditions (1.1), (1.2) we obtain the system

$$A_1(p)(p+\varkappa)\sin(p-1)\alpha + A_2(p)(p-\varkappa)\sin(p+1)\alpha = 2Gu^-(p) \tag{1.8}$$
$$A_1(p)(p+\varkappa)\cos(p-1)\alpha + A_2(p)(p-1)\cos(p+1)\alpha = p^{-1}\sigma^+(p) \tag{1.9}$$
$$A_1(p)p(p+\varkappa)\sin(p-1)\alpha + A_2(p)(p+1)p\sin(p+1)\alpha = 0 \tag{1.10}$$

From (1.9), (1.10) we find

$$A_1(p) = -\sigma^+(p)(p+1)p\sin(p+1)\alpha D_2^{-1}(p),\ A_2(p) = \sigma^+(p)p(p+\varkappa)\sin(p-1)\alpha D_2^{-1}(p) \tag{1.11}$$
$$D_2(p) = -p^2(p+\varkappa)(p\sin 2\alpha + \sin 2p\alpha) \tag{1.12}$$

According to (1.8) the functions $\sigma^+(p)$ and $u^-(p)$ are related by the Wiener-Hopf equation

$$\sigma^+(p) = K(p)u^-(p),\qquad K(p) = D_2(p)D_1^{-1}(p) \tag{1.13}$$
$$D_1(p) = -2G^{-1}p(p+\varkappa)(1-\nu)\sin(p-1)\alpha\sin(p+1)\alpha \tag{1.14}$$

The function $K(p)$ is meromorphic and even. Its poles in the right half-plane $\operatorname{Re} p > 0$ are determined by the formulas

$$p_{k1}^{(1)} = k\pi\alpha^{-1} - 1,\quad p_{k1}^{(2)} = (k-1)\pi\alpha^{-1} + 1\qquad (k = 1, 2, \ldots)$$

and the asymptotics of the large complex zeroes $p_{k2}^{(1)}, p_{k2}^{(2)}$ has the form [3, 4]: $p_{k2}^{(2)} = \bar{p}_{k2}^{(1)}$,

$$p_{k2}^{(1)} = (k - 1/4)\pi\alpha^{-1} + i(2\alpha)^{-1}\ln[(k - 1/4)\pi\alpha^{-1}\sin 2\alpha] + O(k^{-1}\ln k)\quad \text{for}\ \alpha < 1/2\pi \tag{1.15}$$
$$p_{k2}^{(1)} = (k - 3/4)\pi\alpha^{-1} + i(2\alpha)^{-1}\ln[(3/4 - k)\pi\alpha^{-1}\sin 2\alpha] + O(k^{-1}\ln k)\quad \text{for}\ \alpha > 1/2\pi \tag{1.16}$$

For $\alpha \to 1/2\pi$ the pairs of complex zeroes successively go over into pairs of real zeroes; it is then considered that $p_{k2}^{(2)} \geqslant p_{k2}^{(1)}$.

The character of the pole and zero distribution of the function $K(p)$ permits its factorization as follows in the general case:

$$K(p) = K^* p^2 K^-(p)[K^+(p)]^{-1} \tag{1.17}$$

$$K^* = -\frac{G(2\alpha + \sin 2\alpha)}{2(1-\nu)\sin^2\alpha},\qquad K^-(p) = \frac{1}{K^+(-p)} \tag{1.18}$$

$$K^+(p) = \prod_{k=1}^\infty (1 + p/p_{k1}^{(1)})(1 + p/p_{k1}^{(2)})(1 + p/p_{k2}^{(1)})^{-1}(1 + p/p_{k2}^{(2)})^{-1} \tag{1.19}$$

For $\alpha = 1/2\pi$ another factorization is expedient

$$K(p) = K^* p^2 K^-(p)\,[K^+(p)]^{-1}, \qquad K^* = -{}^1\!/_2 G\,(1-\nu)^{-1}$$
$$K^+(p) = [K^-(-p)]^{-1} = \Gamma\,(1 + {}^1\!/_2 p)\,\Gamma^{-1}\,({}^1\!/_2 + {}^1\!/_2 p)$$

The foundation for the absolute and uniform convergence of the infinite product (1.19) and its asymptotic behavior in the right half-plane $\operatorname{Re} p > \operatorname{Re}\lambda$, of the form

$$K^+(p) \sim (\sin\alpha)^{-1}\sqrt{(\alpha + {}^1\!/_2 \sin 2\alpha)p} \quad \text{for} \quad p \to \infty \tag{1.20}$$

are easily obtained by following [4]. The author of [3] first made a detailed investigation of the questions associated with the exact factorization of the type (1.17) - (1.19). The structure of the product (1.19) is such that for p^* real and positive the following inequalities hold:

$$K^+(p^*) > 0, \qquad K^-(-p^*) > 0 \tag{1.21}$$

Taking account of the regularity of the functions with superscripts plus and minus, respectively, in the half-planes $\operatorname{Re} p > \operatorname{Re}\lambda$ and $\operatorname{Re} p < 0$, we obtain a solution of (1.13) on the basis of the condition (1.3), the estimate (1.20) and the generalized Liouville theorem [5]

$$\sigma^+(p) = \gamma_1\,[K^+(p)]^{-1} \tag{1.22}$$

Substituting (1.22) into (1.11) and then into (1.6) and (1.4), we find a symmetric solution of the problem (1.1) - (1.3)

$$u_r = \frac{1}{4\pi i G}\int_L B(p)\,[(p+\varkappa)\sin(p-1)\alpha\cos(p+1)\theta - (p+1)\sin(p+1)\alpha\cos(p-1)\theta]\frac{dp}{r^p}$$

$$u_\theta = \frac{1}{4\pi i G}\int_L B(p)\,[(p-\varkappa)\sin(p-1)\alpha\sin(p+1)\theta - (p+1)\sin(p+1)\alpha\sin(p-1)\theta]\frac{dp}{r^p}$$

$$\tag{1.23}$$

$$\tau_{r\theta} = \frac{1}{2\pi i} B(p)\,p\,(p+1)\,[\sin(p+1)\alpha\sin(p-1)\theta - \sin(p-1)\alpha\sin(p+1)\theta]\frac{dp}{r^{p+1}}$$

$$\sigma_\theta = \frac{1}{2\pi i}\int_L B(p)\,p\,[(p-1)\sin(p-1)\alpha\cos(p+1)\theta - (p+1)\sin(p+1)\alpha\cos(p-1)\theta]\times$$
$$\times\frac{dp}{r^{p+1}}$$

$$\sigma_r = \frac{1}{2\pi i}\int_L B(p)\,p\,[(p+1)\sin(p+1)\alpha\cos(p-1)\theta - (p+3)\sin(p-1)\alpha\cos(p+1)\theta]\times$$
$$\times\frac{dp}{r^{p+1}}$$

$$B(p) = \gamma_1 p\,(p+\varkappa)\,[K^+(p)\,D_2(p)]^{-1} = \gamma_1 p^{-1}(p+\varkappa)\,[K^* K^-(p)\,D_1(p)]^{-1} \tag{1.24}$$

According to the first part of the identity (1.24) for $r > 1$ the integral (1.23) can be decomposed into a series of residues taken at the zeroes of the function $D_2(p)$ from the half-plane $\operatorname{Re} p > \operatorname{Re}\lambda$. The stresses determined by all the zeroes except $p=0$ are hence self-balanced. The residues at the pole $p=0$ connects the quantities P and γ_1 by means of the relationship

$$\gamma_1 = -P(2\sin\alpha)^{-1} \tag{1.25}$$

It will yield the growth of the displacements at infinity

$$\frac{u_\theta}{\sin\theta} = -\frac{u_r}{\cos\theta} = \frac{P(1-\nu)\ln r}{G(2\alpha + \sin 2\alpha)} + O(1)$$

According to the second part of (1.24), the stresses at the wedge apex are determined by the first members of the expansion of the integrals (1.23) in a series of residues at the negative zeroes of the function $D_1(p)$. For $\alpha < {}^1/_2\pi$ these stresses are compressive by virtue of (1.21), finite and independent of the angle θ

$$\sigma_\theta = \sigma_r = -\frac{P \sin \alpha K^+(1)}{\alpha(2\alpha + \sin 2\alpha)} + O(r^{\pi/\alpha - 2}), \qquad \tau_{r\theta} = O(r^{\pi/\alpha-2}) \tag{1.26}$$

For $\alpha > {}^1/_2\pi$ the stresses at the apex of a wedgelike notch become infinite, where their intensity and orders of growth depend on θ and α:

$$\frac{\sigma_\theta}{\cos(\pi\alpha^{-1}\theta)} = -\frac{\sigma_r}{\cos(\pi\alpha^{-1}\theta)} = \frac{\tau_{r\theta}}{\sin(\pi\alpha^{-1}\theta)} = \frac{P(\alpha - {}^1/_2\pi)\sin\alpha r^{\pi/\alpha-2}}{\alpha(\pi-\alpha)(2\alpha + \sin 2\alpha) K^-(1-\pi/\alpha)} + O(1) \tag{1.27}$$

The relationships (1.27) show that the greatest discontinuous σ_θ and compressive σ_r normal stresses originate on the continuation of the notch axis, while the greatest shear stresses originate on the bisectrices $\theta = \pm {}^1/_2\alpha$ separating the tension from the compression zones. As the angle varies between ${}^1/_2\pi$ and π the order of growth of the stresses increases monotonely from r^0 to r^{-1} as $r \to 0$.

The estimates (1.26), (1.27) diverge from the corresponding formula (2.6) of [6]. For $\alpha = {}^3/_4\pi$ the exponents of r differ by 12%, and for $\alpha < {}^1/_2\pi$ formula (2.6) yields infinite stresses at the wedge apex. The reason for such a discrepancy is possibly that this line $u_\theta = \delta$, describing the boundary of a flat stamp in [6], is a hyperbolic spiral, and it can be taken as a line only in those cases when the flat stamp is sufficiently far from the wedge apex.

This remark also refers to [7] in which it would be correct to substitute the conditions $u_\theta = 0$ $(0 \leqslant r \leqslant a)$ and $u_\theta = + h$ $(b \leqslant r < \infty)$ for $\theta = 0$ in place of the conditions $u_\theta = \pm h(0 \leqslant r \leqslant a)$ and $u_\theta = 0$ $(b \leqslant r < \infty)$ which are incompatible with the condition $u_\theta = 0$ for $\theta = h \pm \alpha$, $r = 0$ in problem (1).

The tensile stresses $\sigma_\theta > 0$, originate for the changes mentioned at the apices of the wedge of problem (2), i.e., the slit edges depart from the smooth plate imbedded therein.

Because of the origination of tension zones, solutions of the mixed problem for an infinite wedge considered in [8] can also not be realized, where the principal stress vector equals zero for $r > 1$ and therefore any zone of compressive stresses under the contact surface can be balanced only by a tensile zone.

Let us find the stress distribution under the edges of the yoke (stamp). Substituting (1.20) and (1.25) into (1.22), we obtain

$$\sigma^+(p) \sim -P\,[2(2\alpha + \sin 2\alpha)\,p]^{-1/2} \quad \text{for } p \to \infty, \quad \operatorname{Re} p > 0 \tag{1.28}$$

From the relationships connecting the asymptotics of the functions with their Mellin transformations [5], and from the estimate (1.28) it follows that these stresses are compressive

$$\sigma_\theta \sim -P\,[2\pi(2\alpha + \sin 2\alpha)(1-r)]^{-1/2} \quad \text{for } r \to 1-0, \quad \theta = \alpha \tag{1.29}$$

2. Skew symmetric problem. Let us first examine the problem with the inhomogeneous boundary condition

$$u_\theta = \delta r \quad \text{for } 0 \leqslant r \leqslant 1, \qquad \sigma_\theta = 0 \quad \text{for } 1 < r < \infty \qquad (\theta = \pm\alpha) \tag{2.1}$$

and conditions (1.2), (1.3). Setting $\Phi_0 = 0$, $\Phi_2 = F_1$, $\Phi_1 = -F_2$ into (1.4), we obtain a system of equations from (2.1) and (1.2)

$$A_2(p)(\varkappa - p)\cos(p+1)\alpha + A_1(p)(\varkappa + p)\cos(p-1)\alpha = 2G[u^-(p) + \delta(p+1)^{-1}]$$
$$A_2(p)(p-1)\sin(p+1)\alpha - A_1(p)(\varkappa + p)\sin(p-1)\alpha = p^{-1}\sigma^+(p) \tag{2.2}$$
$$A_2(p)(p+1)\cos(p+1)\alpha - A_1(p)(\varkappa + p)\cos(p-1)\alpha = 0$$

Hence, the unknowns $A_1(p)$ and $A_2(p)$ are expressed by the formulas

$$A_1(p) = \sigma^+(p)p(p+1)\cos(p+1)\alpha D_4^{-1}(p) \tag{2.3}$$
$$A_2(p) = \sigma^+(p)p(p+\varkappa)\cos(p-1)\alpha D_4^{-1}(p)$$
$$D_4(p) = p^2(p+\varkappa)(p\sin 2\alpha - \sin 2p\alpha)$$

and the Wiener-Hopf equation becomes

$$\sigma^+(p) = N(p)[u^-(p) + \delta(p+1)^{-1}] \tag{2.4}$$
$$N(p) = D_4(p)D_3^{-1}(p),\quad D_3(p) = 2G^{-1}p(p+\varkappa)(1-\nu)\cos(p+1)\alpha\cos(p-1)\alpha$$

For $\alpha \neq {}^1/_2\pi$ the function $N(p)$, which has a double zero at the point $p = 0$, is factorized exactly as the function $K(p)$

$$N(p) = \frac{N^* p^2 N^-(p)}{N^+(p)},\quad N^* = \frac{G(\sin 2\alpha - 2\alpha)}{2(1-\nu)\cos^2\alpha},\quad N^-(p) = \frac{1}{N^+(-p)} \tag{2.5}$$

$$N^+(p) = \prod_{k=1}^{\infty}(1 + p/p_{k3}^{(1)})(1 + p/p_{k3}^{(2)})(1 + p/p_{k4}^{(1)})^{-1}(1 + p/p_{k4}^{(2)})^{-1}$$

Here $p_{k3}^{(1)}, p_{k3}^{(2)}$ and $p_{k4}^{(1)}, p_{k4}^{(2)}$ are the poles and zeroes of the function $N(p)$ in the half-plane $\operatorname{Re} p > 0$.

For $\alpha < {}^1/_2\pi$ and $\alpha > {}^1/_2\pi$ the asymptotics of the large complex zeroes $p_{k4}^{(1)}$ is determined by (1.16) and (1.15) respectively, $p_{k4}^{(2)} = -\bar{p}_{k4}^{(1)}$. The formulas for the poles are

$$p_{k3}^{(1)} = p_{k3}^{(2)} - 2 = (k - {}^1/_2)\pi\alpha^{-1} - 1 \quad \text{for } \alpha < {}^1/_2\pi \quad (k = 1, 2, \ldots)$$
$$p_{k3}^{(1)} = (k + {}^1/_2)\pi\alpha^{-1} - 1,\ p_{k3}^{(2)} = (k - {}^3/_2)\pi\alpha^{-1} + 1 \quad \text{for } \alpha > {}^1/_2\pi \quad (k = 1, 2, \ldots)$$

For $\alpha = {}^1/_2\pi$, $N(0) \neq 0$, hence the function $N(p)$ is factorized into

$$N(p) = \pi N(0)N^-(p)[N^+(p)]^{-1},\qquad N(0) = 2G\pi^{-1}(1-\nu)^{-1} \tag{2.6}$$
$$N^+(p) = [N^-(-p)]^{-1} = \Gamma({}^1/_2 + {}^1/_2 p)\Gamma^{-1}(1 + {}^1/_2 p)$$

From (2.4), we obtain by the usual means

$$\sigma^+(p) = [\gamma_2 + \delta(p+1)^{-1}N^-(-1)N^*][N^+(p)]^{-1} \quad \text{for } \alpha \neq {}^1/_2\pi \tag{2.7}$$
$$\sigma^+(p) = \delta(p+1)^{-1}\pi N^-(-1)(0)[N^+(p)]^{-1} \quad \text{for } \alpha = {}^1/_2\pi$$

Substituting (2.3) into (1.6) and (1.4) we find the skew-symmetric solution of the problem (1.1) - (1.3) in which rotation of the wedge induced by condition (2.1) is taken into account in u_θ by the member δr:

$$u_r = \frac{1}{4\pi i G}\int_L C(p)[(p+\varkappa)\cos(p-1)\alpha\sin(p+1)\theta - (p+1)\cos(p+1)\alpha\sin(p-1)\theta]\times$$
$$\times r^{-p}dp$$
$$u_\theta = -\delta r + \frac{1}{4\pi i G}\int_L C(p)[(\varkappa - p)\cos(p-1)\alpha\cos(p+1)\theta +$$
$$+ (p+1)\cos(p+1)\alpha\cos(p-1)\theta]r^{-p}dp \tag{2.8}$$
$$\tau_{r\theta} = \frac{1}{2\pi i}\int_L C(p)p(p+1)[\cos(p-1)\alpha\cos(p+1)\theta - \cos(p+1)\alpha\cos(p-1)\theta]r^{-p-1}dp$$

$$\sigma_\theta = \frac{1}{2\pi i}\int_L C(p)\,p\,[(p-1)\cos(p-1)\alpha\sin(p+1)\theta - (p+1)\cos(p+1)\alpha\sin(p-1)\theta]\,r^{-p-1}dp$$

$$\sigma_r = \frac{1}{2\pi i}\int_L C(p)\,[(p+1)\cos(p+1)\alpha\sin(p-1)\theta - (p+3)\cos(p-1)\alpha\sin(p+1)\theta]\,r^{-p-1}dp$$

$$C(p) = p(p+\varkappa)\,\sigma^+(p)\,D_4^{-1}(p) = p(p+\varkappa)\,[u^-(p)+\delta(p+1)^{-1}]\,D_3^{-1}(p)$$

Evaluating the residues of the integrands of (2.8) at the poles $p = 0$ and $p = 1$, we obtain from the equilibrium equations

$$\gamma_2 = -\frac{Q}{2\cos\alpha} - MN^+(1), \qquad \delta = \frac{N^+(1)}{N^*}\left[\frac{Q}{\cos\alpha} + MN^+(1)\right] \quad \text{for } \alpha \neq {}^1/_2\pi$$

$$\delta = -2M(1-\nu)G^{-1}\pi^{-1} \qquad \text{for } \alpha = {}^1/_2\pi$$

The displacements at infinity and the stresses at the wedge apex and under the edges of the yoke (stamp) are found by analogy with the previous section. For $r \to \infty$ and $\alpha \neq {}^1/_2\pi$ we have

$$u_\theta \sim -\delta r + O(\ln r), \qquad u_r \sim \frac{(1-\nu)\sin\theta}{G(\sin 2\alpha -} \tag{2.9}$$

For $r \to 0$ and $\alpha < {}^1/_2\pi$, taking into account that the integrands of (2.8) have eliminable singularities at the point $p = -1$ and the first pole is at the point $p = p_{13}^{(1)}$, we obtain

$$-\frac{\sigma_\theta}{\sin\beta\theta} = \frac{\sigma_r}{\sin\beta\theta} = \frac{\tau_{r\theta}}{\cos\beta\theta} = -\frac{r^{\beta-2}\cos\alpha N^+(\beta-1)}{\alpha(2\alpha-\sin 2\alpha)}\left[MN^+(1)\cos\alpha + \frac{Q\pi}{2\pi-4\alpha}\right] + O(r^\beta), \qquad \beta = {}^1/_2\pi\alpha^{-1} \tag{2.10}$$

Therefore, the stresses at the wedge apex are infinite for $\alpha > {}^1/_4\pi$. For $r \to 0$ and $\alpha > {}^1/_2\pi$ the residues at the pole $p = p_{13}^{(2)}$ yield

$$\frac{\sigma_\theta}{(4\alpha-\pi)\sin\beta\theta} = -\frac{\sigma_r}{(4\alpha+\pi)\sin\beta\theta} = \frac{\tau_{r\theta}}{\pi\cos\beta\theta} = \frac{r^{-\beta}\cos\alpha N^+(1-\beta)}{\pi\alpha(2\alpha-\sin 2\alpha)}\left[\frac{Q(4\alpha-\pi)}{4\alpha-2\pi} + MN^+(1)\cos\alpha\right] + O(r^{3\beta-2}) \tag{2.11}$$

For $r \to 1-0$, $\theta = \alpha$, $\alpha \neq {}^1/_2\pi$ we obtain

$$\sigma_\theta \sim -\frac{|\cos\alpha|}{\sqrt{\pi(2\alpha-\sin 2\alpha)(1-r)}}\left[2MN^+(1) + \frac{Q}{\cos\alpha}\right] \tag{2.12}$$

3. To utilize the considered solutions in contact problems it is necessary that the normal stresses at the wedge apex and under the yoke (stamp) edges not be tensile, i.e., the condition

$$\sigma_\theta \leqslant 0 \quad \text{for } \theta = \pm\alpha, \quad r \to 0 \text{ and } r \to 1-0 \tag{3.1}$$

must be satisfied.

As formulas (1.26), (1.27) and (1.29) show, the symmetric solution satisfies this condition for all α. The skew symmetric solution evidently always has a tension zone on the line of contact and cannot be realized separately.

According to (1.26), (1.27) and (2.10), (2.11) the stresses at the wedge apex are in the general case

$$\sigma_\theta = s_1(\alpha)\,r^{t_1(\alpha)} \pm s_2(\alpha)\,r^{t_2(\alpha)} \pm s_3(\alpha)\,r^{t_3(\alpha)} \quad \text{for } \theta = \pm\alpha \tag{3.2}$$

Here the first member characterizes the symmetric part in which $s_1(\alpha) < 0$ for $P > 0$, the second and third members are skew symmetric. It is easy to verify that $t_1(\alpha) < t_3(\alpha)$ for all α, while $t_1(\alpha) < t_2(\alpha)$ in the intervals $(0, {}^1/_4\pi)$ and $({}^3/_4\pi, \pi)$, and $t_1(\alpha) > t_2(\alpha)$ in the interval $({}^1/_4\pi, {}^3/_4\pi)$.

Therefore, if $0 < \alpha < {}^1/_4\pi$ or ${}^3/_4\pi < \alpha < \pi$ then for any values of Q and M and for $P > 0$ there is an r_1 such that compressive stresses $\sigma_\theta < 0$ originate in the segments $0 \leqslant r < r_1$, $\theta = \pm\alpha$. To satisfy the remaining portion of condition (3.1), it is necessary and sufficient that the inequality

$$P \sqrt{2\alpha - \sin 2\alpha} > |\, 2MN^+(1)\cos\alpha + Q \,|\, \sqrt{4\alpha + 2\sin 2\alpha} \tag{3.3}$$

be satisfied by virtue of the estimates (1.29) and (2.12).

For ${}^1/_4\pi < \alpha < {}^3/_4\pi$ condition (3.1) holds, according to (2.10), (2.11) and (3.2) only when $s_2(\alpha) = 0$ together with condition (3.3), i.e. when the force and moment are connected by the equalities

$$Q\pi = M(4\alpha - 2\pi)N^+(1)\cos\alpha \quad \text{for } {}^1/_4\pi < \alpha < {}^1/_2\pi$$
$$Q(\pi - 4\alpha) = M(4\alpha - 2\pi)N^+(1)\cos\alpha \quad \text{for } {}^1/_2 < \alpha < {}^3/_4\pi$$

To assure strict adherence to these equalities in practice is impossible. Hence, it is correct to consider that for ${}^1/_4\pi < \alpha < {}^1/_2\pi$ and for ${}^1/_2\pi < \alpha < {}^3/_4\pi$ the solution (1.2) is mechanically unrealizable even in the symmetric case for the yoke (stamp) will depart from an elastic wedge for the smallest violation of symmetry.

The question of the sufficiency of the condition (3.1) for compact abutment of the contacting surfaces requires still another approach and will be examined separately.

BIBLIOGRAPHY

1. Nuller, B. M., Contact problems for an elastic semi-infinite cylinder. PMM Vol. 34, №4, 1970.
2. Ufliand, Ia. S., Integral Transforms in Problems of Elasticity Theory. Moscow-Leningrad, Akad. Nauk SSSR Press, 1963.
3. Al'perin, I. G., Stresses in an infinite strip uniformly compressed over half the length. Uchen. Zap. Khar'kov Univ., №20, 1950.
4. Nuller, B. M., Contact problem for an elastic infinite cone. PMM Vol. 34, №2 1970.
5. Noble, B., Application of the Wiener-Hopf Method to Solve Partial Differential Equations (Russian Translation). Moscow, IIL, 1962.
6. Aleksandrov, V. M., Contact problems for an elastic wedge. Inzh. Zh., Mekh. Tverd. Tela, №2, 1967.
7. Smetanin, B. I., Splitting of an infinite elastic wedge. PMM Vol. 33, №5, 1969.
8. Matczynski, M., Elastic wedge with discontinuous boundary conditions. Arch. Mech. Stos. Vol. 15, №6, 1963.

Translated by M. D. F.